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TWO APPROACHES FOR OPTION PRICING UNDER ILLIQUIDITY[1]

The paper focuses on option pricing under unusual behaviour of the market, when the price may not be changed for some time what is quite a common situation on the modern financial markets. There are some patterns that can cause permanent price gaps to form and lead to illiquidity. For example, global changes that have a negative impact on financial activity, or a small number of market participants, or the market is quite young and is just in the process of developing, etc.

In the paper discrete and continuous time approaches for modelling market with illiquidity and evaluation option pricing were considered.

Trinomial discrete time model improves upon the binomial model by allowing a stock price not only to move up, down but stay the same with certain probabilities, what is a desirable feature for the illiquid modelling. In the paper parameters for real financial data were identified and the backward induction algorithm for building call option price trinomial tree was applied.

Subdiffusive continuous time model allows successfully apply the physical models for describing the trapping events to model financial data stagnation's periods. In this paper the Inverse Gaussian process IG was proposed as a subordinator for the subdiffusive modelling of illiquidity and option pricing. The simulation of the trajectories for subordinator, inverse subordinator and subdiffusive GBM were performed. The Monte Carlo method for option evaluation was applied.

Our aim was not only to compare these two models each with other, but also to show that both models adequately describe the illiquid market and can be used for option pricing on this market. For this purpose absolute relative percentage (ARPE) and root mean squared error (RMSE) for both models were computed and analysed.

Thanks to the proposed approaches, the investor gets a tools, which allows him to take into account the illiquidity.

Keywords: subdiffusion models, subordinator, inverse subordinator, hitting time, trinomial tree model.

Intoduction

Analysis of different financial markets shows that during global crises that have a negative impact on financial activity we can observe some kinds of risky assets which have the periods in their dynamic without change. Such behavior is typical for emerging markets with low number of transactions, for interest rate markets and for commodity markets. So for these markets the problem of evaluating fair price of derivative instruments on stocks have become extremely important.

The classical diffusion models for continuous time like Black-Scholes-Merton (B-S) and its discrete variant - binomial tree model of Cox-Ross-Rubinstein (C-R-R) [2] are incapable of adequately modelling illiquidity for real-life asset dynamic and evaluate derivatives. This happens because classical binomial C-R-R model allows a stock price only to move up or down and do not take into account the stagnation periods. In benchmark B-S model Brownian motions is perpetually moving and we can not use it for modeling periods with motionless stock returns too.

In order to overcome this difficulty for discrete-time approach was considered the trinomial tree model. This model improves upon the binomial model by allowing a stock price not only to move up or down, but stay the same with certain probabilities, what is a desirable properties for the illiquid modelling.

For continuous-time approach one can notice, that the constant periods of stagnation in financial processes are analogous in nature to the trapping events of the subdiffusive particle. Therefore, the physical models of subdiffusion can be successfully applied to describe financial data. See for example paper [6], where option pricing was proposed in fractional jumpвЂ“diffusion model, papers [7] for Black-Scholes formula and [8], [14] for Bachelier

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model in subdiffusive regime.

The aim of the work was to consider two different approaches for modelling market with stagnation periods: to apply trinomial tree model and propose IG prosess as a subordinator for subdiffusive model.

The paper is organized as follows. In the next section we remind what is trinomial tree model and how we can apply it to find fair option price for real historical data. This section is based on the papers [1], [3], [4], [5], where different types of trinomial tree models are presented. We show how model parameters for real financial data can be identified and the backward induction algorithm for building call option price trinomial tree can be applied.

In the third section we consider IG process as subordinator of subdiffusive GBM and its properties. The simulation of the trajectories for subordinator, inverse subordinator and subdiffusive GBM were performed. Also we describe Monte Carlo option pricing techniques for this case.

Forth section contents some numerical results for real financial data, absolute relative percentage (ARPE) and root mean squared errors (RMSE) for both models and its comparison.

Trinomial tree for modelling of illiquidity

Trinomial tree parameters setting. As we mentioned above, Ross-Cox-Rubinstein binomial tree model [2] is incapable of adequately modelling illiquidity for real-life asset dynamic and for evaluating derivatives because this model allows a stock price only to move up or down. A more advanced model that can be used for describing of the stagnation's periods is the trinomial tree model. This model based on the principle that the stock price may move up, down, or stay the same with a certain probability. This rule is important for modelling of the stagnation's periods.

The general form of the tree is as shown in the Figure below.

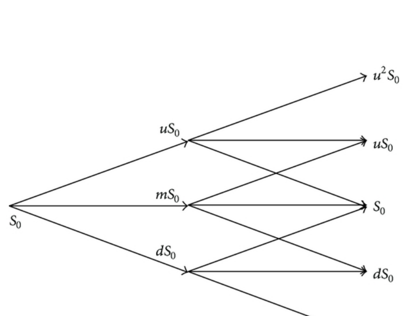

Figure 1. Trinomial tree

Various types of trinomial trees have been proposed in the literature for pricing financial derivatives. See for examples, [3], [4], [5]. As with binomial trees, there is freedom to choose the parameters of a trinomial tree, depending upon what characteristics one wishes to emphasize. For example, one can attempt to match higher moments, or attempt to obtain smooth convergence. The description of the trinomial model in this subsection mostly is based on paper [3]. A trinomial tree is characterized by the following parameters:

u - coefficient of price increase

d - coefficient of price reduction

m - coefficient of price stagnation

p_u- the probability of an increase in the stock price

p_d- the probability of a decrease in the stock price

p_m- the probability of a staying the same in the stock price

We choose the parameters u, d, m to match the volatility σ of the stock price. The step is of length Δt. According to the assumption from [3]:

$$\begin{cases} u = e^{\sigma\sqrt{2\Delta t}} \\ m = 1 \\ d = e^{-\sigma\sqrt{2\Delta t}}. \end{cases} \tag{1}$$

Also one can match the first two moments of our models distribution according to the no arbitrage condition. In a risk-neutral world, the expected return on all assets is equal to the risk-free interest rate (this means that all expected gains are discounted at the rate) and the variance can be expressed as follow [3]:

$$E(S_t) = S_0 e^{r\Delta t} \tag{2}$$

$$var(S_t) = S_0^2 e^{2r\Delta t}(e^{\sigma^2\Delta t} - 1) \tag{3}$$

We equate two values for mathematical expectation (2) and variation (3) to form two equations of the system. Also, using the property that the sum of the probabilities equal to unity, we write down the third equation. So, we got a system of three equations and three unknown variables:

$$\begin{cases} p_u + p_m + p_d = 1 \\ up_u + mp_m + dp_d = e^{r\Delta t} \\ u^2 p_u + m^2 p_m + d^2 p_d - (e^{r\Delta t})^2 = e^{2r\Delta t}(e^{\sigma^2\Delta t} - 1) \end{cases} \tag{4}$$

From this system the probability values for the trinomial model are:

$$\begin{cases} p_u = \dfrac{e^{2r\Delta t}e^{\sigma^2\Delta t} - e^{2r\Delta t}(d+1) + d}{(u-d)(u-1)} \\ p_d = \dfrac{e^{2r\Delta t}e^{\sigma^2\Delta t} - e^{2r\Delta t}(u+1) + u}{(d-u)(d-1)} \\ p_m = 1 - p_u - p_d \end{cases} \tag{5}$$

The above setting (1) for parameters and (5) for probabilities we use in the next sections for option pricing numerical result.

Option pricing for trinomial model . The methodology when pricing options using a trinomial tree is exactly the same as when using a binomial tree. To determine the option price f based on the trinomial tree, the following algorithm is used:

1. Declare and initialize $S(0)$
2. Calculate the jump sizes u, d, m
3. Calculate the transition probabilities

$$p_u, p_d, p_m$$

4. Build the share price tree
5. Calculate the option payoffs at maturity time T, i.e node N:

for the call option

$$|S-K|^{+} = \begin{cases} S-K, & S>K \\ 0, & S \leq K, \end{cases} \tag{6}$$

for the put option

$$|K-S|^{+} = \begin{cases} K-S, & K>S \\ 0, & K \leq S, \end{cases} \tag{7}$$

6. Apply the following backward induction algorithm, where u represents the time position and j the space position

$$f_{u,j} = e^{-r\Delta t}(p_u f_{u+1,j+1} + p_m f_{u+1,j+1} + p_d f_{u+1,j+1}) \tag{8}$$

7. The fair price f of the European call or put option is

$$f = f_{0,0} \tag{9}$$

We apply this algorithm for option pricing for getting numerical results for real financial data with stagnation's periods.

Numerical results for trinomial model. We consider Airbnb company spot price $S_0 = 103.51$ for June 24, 2022. The strike price is $K = 100$ for call options with maturity T is given for ten different dates. The yearly volatility for returns of the underlying asset is computed as $\sigma = 0.5758$, the yearly riskless interest rate is set as $r = 0.16$.

For these input parameters we compute jump sizes and the transition probabilities

$$u = 1.02, m = 1, d = 0.98, \tag{10}$$

$$\begin{cases} p_u = 0.4166 \\ p_d = 0.4169 \\ p_m = 0.1663 \end{cases} \tag{11}$$

and build the share price trinomial tree. The first 5 steps of this tree is demonstrated in the Graph below.

					194.076597
				171.149249	171.149249
			150.930436	150.930436	150.930436
		133.100184	133.100184	133.100184	133.100184
	117.376319	117.376319	117.376319	117.376319	117.376319
103.510002	103.510002	103.510002	103.510002	103.510002	103.510002
	91.28179	91.28179	91.28179	91.28179	91.28179
		80.498164	80.498164	80.498164	80.498164
			70.988468	70.988468	70.988468
				62.602206	62.602206
					55.206659

Figure 2. Trinomial tree for 5 steps

After that we apply the backward induction algorithm and build call option price trinomial tree. See Graph for $T = 5$.

					94.076597
				73.098571	71.149249
			54.791081	52.879758	50.930436
		39.310814	36.960829	35.049506	33.100184
	27.089361	24.631463	22.491543	19.325641	17.376319
18.032227	15.813945	13.800416	10.978134	8.76654	3.510002
	8.205673	6.111252	4.412413	1.684961	0.0
		2.216124	0.808858	0.0	0.0
			0.0	0.0	0.0
				0.0	0.0
					0.0

Figure 3. Tree of pay-off function for 5 steps

The fair price for this call option is $C = 6.1957$.

The results for different times of maturity are demonstrated in the figure 4.

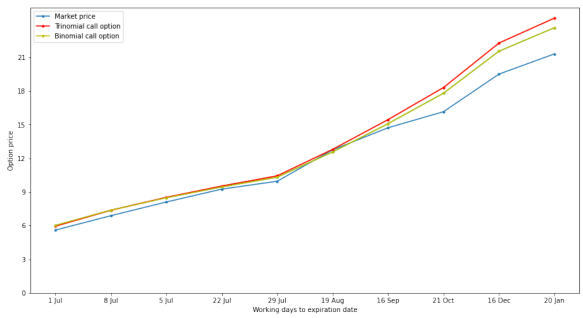

Figure 4. Simulated prices for the binomial and trinomial option pricing models

Subdiffusion for modelling of illiquidity

Subdiffusion processes with IG ***subordinator and its simulation.*** For modelling of illiquidity in continuous case it is useful to apply the subdiffusion process, which is used in statistical physics for describing the trapping events of the subdiffusive particle. In physics, this process usually is described by Fokker-Planck fractal equations.

Equivalent description of subdiffusion there exists in terms of subordination, where the standard diffusion process is time-changed by the so-called inverse subordinator. In this section we consider B-S model and the standard diffusion process

GBM for describing underlying risky asset in subdiffusive regime. For it we replace the calendar time t in classical GBM [7]

$$dX(t) = \left(\mu + \frac{\sigma^2}{2}\right) X(t)dt + \sigma X(t)dB_t, \quad t > 0. \tag{12}$$

with some stochastic process $H(t)$ and obtain subdiffusive GBM

$$dX_{H(t)} = \left(\mu + \frac{\sigma^2}{2}\right) X_{H(t)}dH_t + \sigma X_{H(t)}dB_{H(t)}. \tag{13}$$

In formula (13) $H(t)$ is called the inverse subordinator and it definded as

$$H(t) = inf\,(\tau > 0 : G(\tau) > t)\,.$$

The inverse subordinator $H(t)$ is also called a "hitting time" and is interpreted as the time of first reaching a certain price, which may not change for some time. By construction, the inverted process may be constant. Therefore, any process subordinated by $H(t)$ exhibits motionless periods.

The difinition (3.1) of the inverse subordinator is based on the use of some other random process called a subordinator $G(t)$.

The subordinator $G(t)$, in its turn, is generally a non-decreasing stochastic process with stationary independent increments with right continuous left limits sample paths.

Many types of subordinators such as α-stable, tempered-stable, Gamma, Poisson and other have been already applied for different subdiffusive models of illiquidity (see for example [6], [7], [8], [14]).

In this paper we propose to take the Inverse Gaussian process IG as a subordinator for the subdiffusive modelling. The $G(t)$ process is a non-decreasing Levy process (i.e., process with stationary independent increments), where the increments $G(t+s) - G(s)$ follow the inverse Gaussian $G(\delta t, \gamma)$ distribution with probabilities density function (PDF) with parameters γ and δ (see for example [9]):

$$g(x,t,\gamma,\delta) = \frac{\delta t}{\sqrt{2\pi x^3}} e^{\delta\gamma t - (\delta^2 t^2/x + \gamma^2 x)/2}, \quad x > 0$$

For the standard IG distribution, where $\gamma = \delta = 1$ the PDF will be

$$f(x,t) = \frac{t}{\sqrt{2\pi x^3}} \exp\left(-\frac{(x-t)^2}{2x}\right), \quad x > 0,$$

Then for any moment t we have $E(G(t)) = t$, $var(G(t)) = t$.

The tail probability for $G(\delta t, \gamma)$ is studied in [9] and equals

$$P(G(t) > x) \sim \sqrt{\frac{2}{\pi}} \frac{\delta t}{\gamma^2} e^{\gamma\delta t} x^{-3/2} e^{-(\gamma^2/2)x}, \quad x \to \infty.$$

The q-th order moments of the $G(\delta t, \gamma)$ are given by

$$EG^q(t) = \sqrt{\frac{2}{\pi}\delta} \left(\frac{\delta}{\gamma}\right)^{q-1/2} t^{q+1/2} e^{\delta\gamma t} K_{q-1/2}(\delta\gamma t),$$

where $K_q(\omega)$ is the modified Bessel function of the third kind with index q, defined in [9].

The algorithm of the simulation of the IG process $G(t)$ for time points $t_1 = \frac{1}{n}, t_2 = \frac{2}{n}, ..., t_n = 1$ can be presented into the following steps [9]:

1. For $i = 1, 2, ..., n$ and $dt = 1/n$ we generate n independent identically distributed inverse Gaussian variables F_i assuming $\gamma = \Delta = 1$

a) Generate a standard normal random variable N.

b) Assign $X = N^2$.

c) Assign $Y = dt + \frac{X}{2} + \frac{1}{2} * \sqrt{4dt}$.

d) Generate a uniform $[0, 1]$ random variable U.

e) If $U \leq \frac{dt}{dt+Y}$ return Y; otherwise return $\frac{(dt)^2}{Y}$

2. Assign $G(t_0) = 0$ and $G(t_i) = \sum_{j=1}^{i} F_j, i = 1, 2, ..., n$

3. $G(t_1), G(t_2), ..., G(t_n)$ are n simulated values of the IG process at times $t_1, t_2, ..., t_n$ respectively.

The simulation of the trajectory $G(t)$ is demonstrated below on Figure 6.

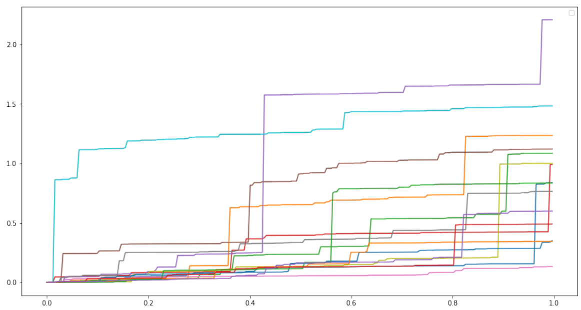

Figure 5. Simulation of the IG process trajectories

The inverse subordinator $H(t)$ is also called a hitting time or stochastic clock defined by (3.1) is the inverse to the IG process. The IIG process was studied in [9], where were found as q-th order moments of the $IIG(\delta t, \gamma)$ as its tail behaviour.

In order to simulate the approximate trajectory inverse subordinator $H(t)$, we define $H(\Delta t)$ with the step length Δ as follows [9]:

$$H_\Delta(t) = [\min\{n \in N : G(\Delta n) > t\} - 1]\Delta, \quad n = 1, 2, \ldots \tag{14}$$

where Δ is the step length and $G(\Delta n)$ is the value of the Inverse Gaussian process $G(t)$ evaluated at n.

The simulation of the trajectory $H(t)$ is demonstrated on Figure 7.

For simulation of the trajectory subdiffusive GBM $X(t)$ we remind that the Ito equation allows modeling the time dynamics of an arbitrary stochastic process by means of the *iterative scheme*[15]:

$$x_{k+1} = x_k + a\left(x_k, t_k\right) \triangle t + b(x_k, t_k) \sqrt{\triangle t} \varepsilon_k. \quad (15)$$

In paper [16] were considered iterative schemes for fractal activity time processes with inverted gamma subordinator. For modeling stochastic subdiffusive GBM we propose the next iterative scheme

$$x_{k+1} = x_k + \mu x_k \Delta H(t) + \sigma x_k \sqrt{\Delta H(t)} \varepsilon_k, \quad (16)$$

where ε is white noise with normal standard distribution, $\Delta H(t)$ have IIG distribution.

The simulation of the trajectory $X(t)$ according (16) is demonstrated on the Figure 6.

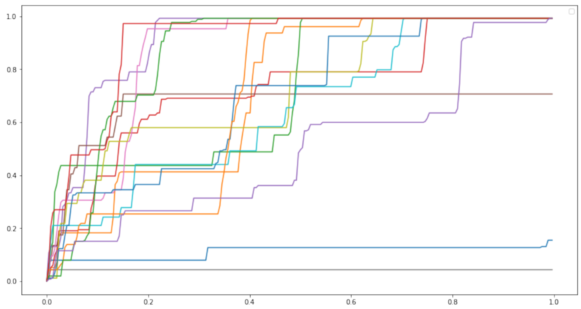

Figure 6. Simulation of the inverse to the IG process trajectories

Meanwhile, the trajectory for the subdiffusion GBM with the inverse to the IG process is demonstrated on the Figure 7.

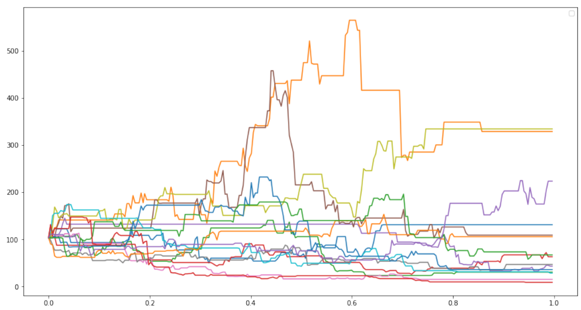

Figure 7. Simulation of the subdiffused Geometric Brownian motion with inversed IG subordinator

Monte Carlo method for option pricing in subdiffusion Black - Scholes model. The fair price of the European call option in the non fractional B-S model (12) is given by:

$$C(S, K, T, r, \sigma) = N(d_1)S - N(d_2)Ke^{-rT} \quad (17)$$

with

$$d_1 = \frac{\log \frac{S_0}{K} + rT + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \quad (18)$$

$$d_2 = \frac{\log \frac{S_0}{K} + rT - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \quad (19)$$

are both functions of five parameters: T, K, S_0, r, σ, and $N(\cdot)$ is a standard normal cumulative distribution function, T is time to maturity (in years), r is interest rate and σ is volatility.

Consider a time-changed version of the B-S model, where the underlying risky assets follow (13). Then, as were shown in [7] the market model is arbitrage-free and incomplete and the corresponding fair price of the European call option in subdiffusive regime [7] is

$$C_{sub}\left(S, K, T, \sigma\right) = \left\langle C\left(S, K, H(T), \sigma\right)\right\rangle$$
$$= \int_0^\infty C\left(S, K, x, \sigma\right) g(x, T) dx \quad (20)$$

Here, $g(x, T)$ is the PDF of $H(T)$ and $C(S, K, T, \sigma)$ is given by (17).

It is worth to mention, that the proof of formula (20) for fair price is based on the common ideas for changed time models, see for examples proof in [11] for Student model with FAT or for Student-like FAT in [10] and their applications in [13], [12].

There are two ways of finding the values of the price $C(\cdot)$. One is to calculate $C(\cdot)$ by approximating the integral in (20). However, this can be performed in cases, where $g(x, T)$ is known exactly.

The other way is to find $C(\cdot)$ by using the Monte-Carlo method. One simulates the trajectories for the inverse subordinator on the interval $[0, T]$ by the approximation scheme (14). Then, one obtains the fair price as an estimation of the expected value for simulated prices where the inverse subordinator stands for calendar time T in (20)

$$C_{sub}(S, K, T, r, \sigma) = \langle C(S, K, H(T), \sigma)\rangle$$
$$= \frac{1}{n}\sum_{i=1}^{n} C(S, K, H_i(T), \sigma), \quad (21)$$

where $C(S, K, T, \sigma)$ is taken from Black-Scholes option pricing formula (17).

One can see the applying of the Monte-Carlo method for option pricing in subdiffusive models, for example, in the papers [7], [8], [14].

Numerical results for subdiffusive Black--Scholes model . For the company "Airbnb" the input parameters are: $S = 103.51$, $K = 100$, $r = 0.16$, $\sigma_{diff} = 0.5758$ for the diffusion model (see section 2.3 above).

First we simulate N trajectories of subordinator $G(t)$, that is a process of independent stationary increments having IG distribution.

After that we simulate N values of the inverse IG subordinator $H(T)$ for every given time to maturity T and calculate N option price values, using Black-Scholes option pricing formula (17).

Then find the fair price as a mean for N scenarios, obtained in the previous step according (21).

The results are presented in the graphic shape in Fig. 8.

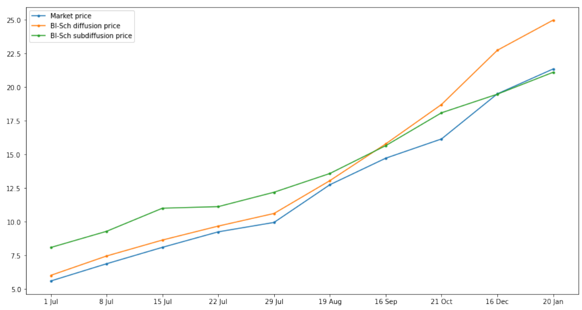

Figure 8. Simulated prices for the diffusive and subdiffusive B-S models

As we can see from graphics in Fig. 8., the diffusive option pricing model shows better results on the short-term period, while the subdiffusive model is more effective on the long-term perspective.

For more detail we need to compute and compare the estimation errors.

Comparison of the two models

In this section we compare numerical results for AIRBNB company for two proposed models. It is a trinomial tree model and subdiffusive B-S model with IG subordinator.

Our aim is not only to compare these two models each with other, but also to show that both models adequately describe the illiquid market.

In Fig. 9 we compare the subdiffusive B-S formula for European call options with the classical one and with option pricing using trinomial tree model. We estimated the values of subdiffusive B-S formula using Monte Carlo methods based on the above described simulation procedure.

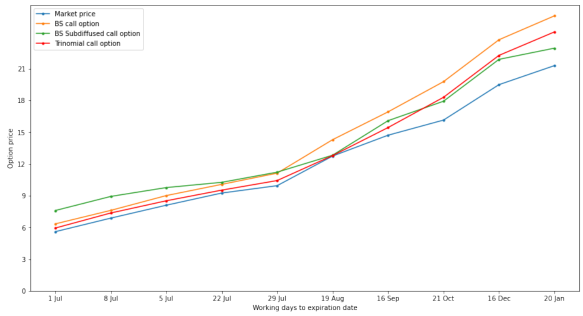

Figure 9. Comparison of the trinomial model and the B-S subdiffusive approach for the call option pricing

To compare numerical results we use absolute relative percentage (ARPE) and root mean squared error (RMSE):

$$ARPE = \frac{|x(t_k) - x_{exact}(t_k)|}{x_{exact}(t_k)} \tag{22}$$

$$RMSE = \sqrt{\frac{1}{n}\Sigma_{i=1}^{n}\left(\frac{x_i - x_{exact_i}}{\sigma_i}\right)^2} \tag{23}$$

It is worth to mention, in econometrics, the root mean squared error (RMSE) (22) is a key criterion for model selection. The mean squared error indicates the mean squared deviation between the forecast and the outcome. It sums the squared bias and the variance of the estimator.

The advantage of the ARPE (23) relatively to the RMSE measure is that it gives a percentage value of the pricing error.

Therefore, if we use both these errors it provides more insight into the economic significance of performance differences.

	RMSE
B-S	1.82
B-S Subdiffusion	1.85
Trinomial model	1.54

Table 1. The RMS errors for diffusion, subdiffusion and trinomial models regarding to the market price

Conclusion

In the paper two different approaches for modelling market with stagnation periods were considered. We apply well-known trinomial tree model in discrete time case and propose subdiffusive model with IG subordinator in continuous time case.

For the option pricing the backward induction algorithm trinomial tree model was used. In the continuous time case Monte-Carlo method was proposed.

The programmed model can be used to valuate option price by several different methods and it can help to make decision.

To compare numerical results we used absolute relative percentage (ARPE) and root mean squared errors (RMSE).

In the framework of the paper we compared option prising results in situation when strike price K was fixed (in the money), while time to maturity T were changing.

If we compare classical B-S model with subdiffusive one, the results show that the diffusive option pricing B-S model shows better results on the short-term period, while the subdiffusive model is more effective on the long-term perspective. Meanwile RMSE is bigger for proposed subdiffusive model then for classical B-S one. Comparing subdiffusive B-S model with trinomial one we assume that trinomial model has the smallest RMS error.

In the future we are going to examine the ARP pricing errors of the proposed option pricing models in more detail (see paper [17]) and consider the pricing errors as a regression on the time to maturity T(in years), the moneyness of the option, and a binary variable that is set to unity, if the option is a call and to zero in the case of a put. This can indicate a level of explanatory value of moneyness, maturity and the put-call dummy in the model.

Our next step is to apply the procedure of calculating value-at-risk in the proposed model (with IG subordinator) and analyze it for different types of investor portfolios like in the papers [17], [18].

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ДВА ПІДХОДИ ДО ЦІНОУТВОРЕННЯ ОПЦІОНІВ В УМОВАХ НЕЛІКВІДНОСТІ

Статтю присвячено ціноутворенню опціонів в умовах неліквідності, коли ціна на ринку може не змінюватися протягом деякого часу, що є досить поширеною ситуацією на сучасних фінансових ринках (наприклад, глобальні зміни, які негативно впливають на фінансову діяльність, або невелика кількість учасників ринку, або ринок, що тільки розвивається, тощо).

У статті розглянуто дискретний і неперервний підходи для моделювання та ціноутворення опціонів в умовах ринку з неліквідністю.

Для дискретного часу було обрано триноміальну модель, що вдосконалює біноміальну, дозволяючи ціні акцій не тільки рухатися вгору, вниз, але й залишатися незмінною з певною ймовірністю, що є бажаною властивістю моделювання в умовах неліквідності. У статті були визначені параметри триноміальної моделі для реальних фінансових даних і застосовано алгоритм зворотної індукції для оцінки ціни кол-опціону.

Для неперервного часу для моделювання періодів стагнації фінансових даних успішно застосовується субдифузійна модель,що з'явилася для опису подій захоплення фізичних частинок. У цій статті був запропонований обернений гаусівський процес як субординатор для субдифузійного моделювання неліквідності та ціни опціонів. Виконано симуляцію траєкторій для субординатора, оберненого субординатора та субдифузійного ГБМ. Для оцінки опціонів застосовано метод Монте-Карло.

Нашою метою було не тільки порівняти ці дві моделі, а й показати, що обидві моделі адекватно описують неліквідний ринок і можуть бути використані для ціноутворення опціонів на цьому ринку. Для цього було розраховано та проаналізовано абсолютні відносні (ARPE) і середньоквадратичні помилки (RMSE) для обох моделей.

Завдяки запропонованим підходам інвестор отримує інструментарій, який дає змогу врахувати неліквідність.

Ключові слова: субдифузійна модель, субординатор, обернений субординатор, час попадання, триноміальна модель.

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