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DEVIATION OF THE INTERFACE BETWEEN TWO LIQUID HALF-SPACES WITH SURFACE TENSION: MULTISCALE APPROACH

This paper investigates the deviation of the interface between two semi-infinite liquid media under the influence of surface tension and gravity using a multiscale analysis. The initial-boundary value problem is formulated based on key dimensionless parameters, such as the density ratio and the surface tension coefficient, to describe the generation and propagation of wave packets along the interface. A weakly nonlinear model is employed to examine initial deviations of the interface, enabling the derivation of integral solutions for both linear and nonlinear approximations. The linear approximation captures the fundamental structure of forward and backward waves, while nonlinear corrections account for higher-order effects derived through multiscale expansions. These corrections describe the evolution of the wave packet envelope, highlighting the interplay between dispersion, nonlinearity, and surface tension. Integral expressions are provided for both linear and nonlinear solutions, including those illustrating the role of even and odd initial deviations of the interface. Comparisons between linear and nonlinear approximations emphasize their interconnectedness. The linear model defines the primary wave dynamics, while the nonlinear terms contribute higher harmonics, refining the solutions and facilitating stability analysis. The results reveal significant contributions from higher-order harmonics in determining the dynamics of the interface. Furthermore, the study explores the conditions under which the nonlinear envelope remains stable, including constraints on initial amplitudes to prevent instability. This research opens new perspectives for further analysis of stability and wave dynamics at fluid interfaces using symbolic computations. Potential applications include the study of wave behavior under various geometric configurations and fluid properties. The findings contribute to advancing hydrodynamic wave modeling and establish a foundation for future research in this field.

Keywords: internal waves, initial-boundary value problem, multiscale expansions, surface tension.

Introduction

The study of wave packets along the interface of two semi-infinite fluids forms the basis for solving initial-boundary value problems (IBVPs) related to the generation and evolution of internal waves. These include the transmissibility of wave harmonics and modulational stability, or the so-called Benjamin–Feir instability [1].

Benjamin-Feir instability in hydrodynamics has been widely analyzed, focusing on stabilization mechanisms and extreme wave formation. Segur et al. [2] demonstrated dissipation stabilizes instability for waves with narrow bandwidth, confirmed experimentally; Wu [3] supported this via simulations, while Onorato et al. [4] linked the Benjamin-Feir index to extreme wave probability. Zakharov and Ostrovsky [5] explored nonlinear effects from modulation instability, and El and Hoefer [6] reviewed dispersive shock waves. Armaroli et al. [7] validated wave stabilization under wind-viscosity balance through experiments.

It should be noted that wave propagation in layered fluids has been effectively studied using multiscale methods. Here, we will mention only a few studies in this field. Nayfeh [8] derived an envelope evolution equation (NLS) for waves on fluid interfaces with surface tension. Grimshaw and Pullin [9] examined modulational stability of finite-amplitude interfacial waves, while Selezov et al. [10] investigated nonlinear wave-packet propagation using higher-order multiscale expansions.

This work extends the IBVP for the deviation of the contact surface between two semi-infinite fluids under surface tension, incorporating nonlinear effects and advancing understanding of interfacial wave dynamics.

Statement of the IBVP

Problem statement. This paper investigates the IBVP based on the solutions of problem [8] concerning traveling wave packets of dispersive nature. The following parameters were introduced as the basis for dimensionless quantities: the acceleration due to gravity g, the density ρ_1, and the characteristic surface tension T_0.

The problem of wave packet propagation along the interface $z = \eta(x,t)$ between two fluids of different densities was addressed, with the effects of surface tension T is taken into consideration

$$\begin{aligned} &\Delta\phi_j = 0 \quad \text{in} \quad \Omega_j, \\ &\eta_{,t} - \phi_{j,z} = -\alpha\eta_{,x}\phi_{j,x} \quad \text{at} \quad z = \alpha\eta(x,t), \\ &\phi_{1,t} - \rho\phi_{2,t} + (1-\rho)\eta + 0.5\alpha\left(\nabla\phi_1\right)^2 - 0.5\alpha\rho\left(\nabla\phi_2\right)^2 \\ &\quad - T\left(1 + (\alpha\eta_{,x})^2\right)^{-3/2}\eta_{,xx} = 0 \quad \text{at}\, z = \alpha\eta(x,t), \\ &|\nabla\phi_1| \to 0 \quad \text{at} \quad z \to \pm\infty, \end{aligned} \tag{1}$$

where $\Omega_1 = \{(x,z) : |x| < +\infty, -\infty < z < 0\}$, $\Omega_2 = \{(x,z) : |x| < +\infty, 0 < z < +\infty\}$, $\rho = \rho_2/\rho_1$, $\rho_i (i = 1,2)$ are the densities of fluids in Ω_i, $\alpha = a/l$ is a small parameter characterizing the steepness of the wave, a is the maximum deviation of the contact surface $\eta(x,t)$, and l is the wavelength.

Let the initial condition at $z = 0$ be given as a deviation $F(x)$ of the interface

$$\eta(x,0) = F(x). \tag{2}$$

Preliminary results on traveling wave packets. The result presented in this study is based on previously obtained findings for traveling wave packets derived using the method of multiscale expansions [8]. The results from the aforementioned study, essential for solving the IBVP (1)-(2), are presented below.

According to the method of multiple-scale expansions, the deviation of the interface is represented as a sum of the first harmonics

$$\begin{aligned} \eta(x,t) = &\eta_1(x_0,x_1,x_2,t_0,t_1,t_2) \\ &+ \alpha\eta_2(x_0,x_1,x_2,t_0,t_1,t_2) + \mathrm{O}(\alpha^2), \end{aligned} \tag{3}$$

where $x_n = \alpha^n x$, $t_n = \alpha^n t$ are the spatial and temporal scaling variables.

In the first approximation, the deviation of the contact surface caused by a forward wave η_1^+ is expressed as the sum of the product of the complex envelope $A(x_1,x_2,t_1,t_2)$ and the carrier forward wave $\exp \mathrm{i}(kx_0 - \omega t_0)$ and the product of their conjugates,

$$\eta_1^+ = A\exp(\mathrm{i}(kx_0 - \omega t_0)) + \overline{A}\exp(-\mathrm{i}(kx_0 - \omega t_0)) \tag{4}$$

where, in the linear approximation, the envelope is considered a constant value, as it cannot depend on higher-order scales.

The solvability of the linear approximation problem determines the dispersion relation, which links ω and k; let its two solutions be denoted as $\omega_{1,2} = \pm\omega(k)$.

Here, we present the expression for the second forward harmonic, derived in [8] from the linear problem of the second approximation, in the form

$$\begin{aligned} \eta_2^+ = &\Lambda(k,\omega)A^2\exp(2\mathrm{i}(kx_0 - \omega t_0)) \\ &+ \Lambda(k,\omega)\overline{A}^2\exp(-2\mathrm{i}(kx_0 - \omega t_0)) \end{aligned} \tag{5}$$

where the coefficient $\Lambda(k,\omega)$ satisfies the condition $\Lambda(k,\omega) = \Lambda(k,-\omega)$.

In [8], it is also shown that the envelope A satisfies an evolution equation in the form of a NLS

$$A_{,\zeta} - 0.5\mathrm{i}\omega'' A_{,\xi\xi} = 4\mathrm{i}\alpha^2\omega^{-1}JA^2\overline{A}, \tag{6}$$

where $\xi = x - \omega' t$ and $\zeta = t$, $J(k,\omega)$ is the Benjamin-Feir index which in this system satisfies the condition $J(k,\omega) = J(k,-\omega)$.

The IBVP solution

Linear approximation. Since the hydrodynamic system allows the propagation of only certain types of harmonics (4) and (5), with the frequency ω and wavenumber k linked by the dispersion relation, and the envelope A governed by the evolution equation (6), the problem arises of correctly specifying the initial shape $F(x)$ of the contact surface η within the framework of the weakly nonlinear model (1)-(2). Let the initial position of the contact surface $\eta(x,0)$ in the linear approximation take the form of a certain function $f(x)$

$$\eta_{lin}(x,0) = f(x). \tag{7}$$

On the one hand, the function $f(x)$ can be represented as an integral using the Fourier expansion over the frequency spectrum, followed by synthesis based on this spectrum

$$\begin{aligned} &f(x) = \\ &\Re\left[\int_{-\infty}^{+\infty}\left(\frac{1}{2\pi}\int_{-\infty}^{+\infty} f(\xi)\exp(-\mathrm{i}k\xi)d\xi\right)\exp(\mathrm{i}kx)dk\right] \end{aligned} \tag{8}$$

Considering equations (7) and (8), we have

$$\begin{aligned} &\eta_{lin}(x,0) = \\ &\quad \int_{-\infty}^{+\infty}\Big(a_f(k)\cos(kx) - b_f(k)\sin(kx)\Big)dk, \end{aligned} \tag{9}$$

where

$$a_f(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(\xi)\cos k\xi d\xi, \tag{10}$$

$$b_f(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(\xi)\sin k\xi d\xi. \tag{11}$$

On the other hand, taking into account the dispersion relation solution $\omega_1 = +\omega(k)$, in the linear approximation, the oscillation of the contact surface can be represented as an integral over the wave numbers of the forward wave (4)

$$\eta^{+}_{lin}(x,t) = \int_{-\infty}^{+\infty} \Big(a_{lin}(k)\exp(\mathrm{i}(kx-\omega(k)t)) + \overline{a_{lin}}(k)\exp(-\mathrm{i}(kx-\omega(k)t)) \Big) dk, \quad (12)$$

and for the dispersion relation solution $\omega_2 = -\omega(k)$ corresponding to the backward wave

$$\eta^{-}_{lin}(x,t) = \int_{-\infty}^{+\infty} \Big(a_{lin}(k)\exp(\mathrm{i}(kx+\omega(k)t)) + \overline{a_{lin}}(k)\exp(-\mathrm{i}(kx+\omega(k)t)) \Big) dk \quad (13)$$

where $a_{\rm lin}(k)$ are the unknown coefficients of the linear approximation expansion of the interface deviation, which coincide for the forward $\eta^{+}_{lin}(x,t)$ and backward $\eta^{-}_{lin}(x,t)$ waves due to the homogeneity of liquid media in both directions of wave propagation.

It is obvious that

$$\eta_{lin}(x,t) = \eta^{+}_{lin}(x,t) + \eta^{-}_{lin}(x,t). \quad (14)$$

Next, we obtain $\eta_{lin}(x,0)$ from (14) taking into account (12) and (13)

$$\eta_{lin}(x,0) = 2\int_{-\infty}^{+\infty} \Big(a_{lin}(k)\exp(\mathrm{i}kx) + \overline{a_{lin}}(k)\exp(-\mathrm{i}kx) \Big) dk. \quad (15)$$

Equating the expressions for the initial deviation of the contact surface $\eta_{lin}(x,0)$ from (9) and (15), we obtain

$$a_{lin}(k) = \frac{1}{4}(a_f(k) + \mathrm{i}b_f(k)). \quad (16)$$

Substituting formulas (10) and (11) into (16) we obtain

$$a_{lin}(k) = \frac{1}{8\pi}\int_{-\infty}^{+\infty} f(\xi)\exp(\mathrm{i}k\xi)d\xi, \quad (17)$$

and substituting (17) into (12)-(14) gives the linear approximation of the contact surface deviation in the following integral form

$$\eta_{lin}(x,t) = \quad (18)$$
$$\frac{1}{8\pi}\int_{-\infty}^{+\infty}\Big[\int_{-\infty}^{+\infty} f(\xi)\exp(\mathrm{i}k\xi)d\xi\exp\mathrm{i}(kx-\omega(k)t) + \int_{-\infty}^{+\infty} f(\xi)\exp(-\mathrm{i}k\xi)d\xi\exp(-\mathrm{i}(kx-\omega(k)t))\Big]dk.$$
$$+\frac{1}{8\pi}\int_{-\infty}^{+\infty}\Big[\int_{-\infty}^{+\infty} f(\xi)\exp(\mathrm{i}k\xi)d\xi\exp\mathrm{i}(kx+\omega(k)t) + \int_{-\infty}^{+\infty} f(\xi)\exp(-\mathrm{i}k\xi)d\xi\exp(-\mathrm{i}(kx+\omega(k)t))\Big]dk.$$

or taking into account the formulae (10) and (11)

$$\eta_{lin}(x,t) = \eta^{+}_{lin}(x,t) + \eta^{-}_{lin}(x,t) \quad (19)$$

where

$$\eta^{\pm}_{lin}(x,t) = \frac{1}{2}\int_{-\infty}^{+\infty} \Big(a_f(k)\cos(kx \mp \omega(k)t) - b_f(k)\sin(kx\mp\omega(k)t) \Big) dk. \quad (20)$$

Nonlinear approximation. Let us proceed to derive the nonlinear approximation of the contact surface deviation. To this end, we consider one of the solutions to the evolution equation (6) and write it for both solutions of the dispersion equation

$$A_{\pm}(t,k) = \frac{1}{2}a\exp\left(\pm\mathrm{i}a^2\frac{J(k,\pm\omega(k))}{\omega(k)}t\right), \quad (21)$$

where a is an arbitrary constant determining the amplitude of the envelope, $A_{+}(t,k)$ corresponds to the forward wave with frequency $\omega_1 = +\omega(k)$, and $A_{-}(t,k)$ to the backward wave with frequency $\omega_2 = -\omega(k)$.

Then, for wave packets traveling along the contact surface, taking into account (4) and (5) and also the fact that $J(k,\omega) = J(k,-\omega)$ and $\Lambda(k,\omega) = \Lambda(k,-\omega)$, the contact surface deviation caused by the forward $\eta^{+}_{nl}(x,t,k)$ and backward $\eta^{-}_{nl}(x,t,k)$ waves at the wave number k are expressed as the sum of the harmonics

$$\eta^{\pm}_{nl}(x,t,k) = \quad (22)$$
$$A_{\pm}(t,k)\exp(\mathrm{i}(kx\mp\omega(k)t)) + \overline{A_{\pm}}(t,k)\exp(-\mathrm{i}(kx\mp\omega(k)t)) + \alpha\Lambda(k,\omega(k))\Big(A^2_{\pm}(t,k)\exp(2\mathrm{i}(kx\mp\omega(k)t)) + \overline{A_{+}}^2(t,k)\exp(-2\mathrm{i}(kx\mp\omega(k)t))\Big) + O(\alpha^2).$$

Assume that the coefficient $\frac{1}{2}a$ in the expressions (21) for the envelopes $A_{\pm}(t,k)$ is equal to the complex coefficient $a_{\rm lin}(k)$ and taking into account (16), it can be expressed in terms of the coefficients $a_f(k)$ and $b_f(k)$ of the function $f(x)$ expansion

$$a = \frac{1}{2}\Big(a_f(k) + \mathrm{i}b_f(k)\Big). \quad (23)$$

Let us substitute the expressions for the envelope (21) into (22) taking into account (23). After transformations, we obtain expressions for the nonlinear approximation of the forward $\eta_{nl}^{+}(x,t,k)$ and backward $\eta_{nl}^{-}(x,t,k)$ traveling waves in the form of real-valued expressions

$$\eta_{nl}^{\pm}(x,t,k) = \frac{1}{2}\big[a_f(k)\cos(kx \mp \hat{\omega}t) - b_f(k)\sin(kx \mp \hat{\omega}t)\big] + \frac{\alpha}{8}\Lambda(k,\omega(k))\Big(\big(a_f^2(k) - b_f^2(k)\big)\cos 2(kx \mp \hat{\omega}(k)t) - 2a_f(k)b_f(k)\sin 2(kx \mp \hat{\omega}(k)t)\Big) + \mathrm{O}(\alpha^2), \tag{24}$$

where

$$\hat{\omega}(k) = \omega(k) - a^2\frac{J(k,\omega(k))}{\omega(k)}. \tag{25}$$

Performing the synthesis of the traveling waves (24) by the spectrum of wave numbers, we obtain expressions for the nonlinear approximation of the contact surface deviation $\eta(x,t)$ in the form

$$\eta(x,t) = \eta_1^{+}(x,t) + \eta_1^{-}(x,t) + \alpha\big(\eta_2^{+}(x,t) + \eta_2^{-}(x,t)\big) + \mathrm{O}(\alpha^2), \tag{26}$$

where

$$\eta_1^{\pm}(x,t) = \frac{1}{2}\int_{-\infty}^{+\infty}\Big(a_f(k)\cos(kx \mp \hat{\omega}t) - b_f(k)\sin(kx \mp \hat{\omega}t)\Big)dk, \tag{27}$$

$$\eta_2^{\pm}(x,t) = \frac{1}{8}\int_{-\infty}^{+\infty}\Lambda(k,\omega(k))\Big(\big(a_f^2(k) - b_f^2(k)\big)\cos 2(kx \mp \hat{\omega}(k)t) - 2a_f(k)b_f(k)\sin 2(kx \mp \hat{\omega}(k)t)\Big)dk. \tag{28}$$

The expression (26), obtained for the interface deviation $\eta(x,t)$, contains the nonlinear contribution (27) of the first harmonic $\eta_1^{\pm}(x,t)$, which differs from its linear approximation $\eta_{lin}^{\pm}(x,t)$ in (20). It additionally includes (28) the contribution of the second harmonic $\eta_2^{\pm}(x,t)$. Let us introduce the terms

$$\eta_1(x,t) = \eta_1^{+}(x,t) + \eta_1^{-}(x,t),$$
$$\eta_2(x,t) = \eta_2^{+}(x,t) + \eta_2^{-}(x,t)$$

denoting the contributions of the first and second harmonics to the nonlinear solution.

It should be noted that within the framework of the nonlinear model, the synthesis operation over the spectrum of traveling waves, namely, the first and second harmonics, is mathematically valid, since each of the traveling harmonic waves represents a solution to the linear approximations of the problem at the corresponding order.

Let us now return to the question of the form of the initial contact surface deviation $F(x)$, which in the linear approximation we defined in (7) as some function $f(x)$. From (26)-(28), it is straightforward to obtain

$$F(x) \equiv \eta(x,0) = \int_{-\infty}^{+\infty}\Big[a_f(k)\cos(kx) - b_f(k)\sin(kx) + \frac{\alpha}{4}\Lambda(k,\omega(k))\Big(\big(a_f^2(k) - b_f^2(k)\big)\cos 2(kx) - 2a_f(k)b_f(k)\sin 2(kx)\Big)\Big]dk + \mathrm{O}(\alpha^2). \tag{29}$$

It is evident that

$$F(x) = f(x) + \mathrm{O}(\alpha),$$

i.e., the refined initial contact surface deviation differs from the specified initial deviation in the linear approximation by a small quantity.

Let us consider the stability conditions for the envelope in the form of the solution (21) discussed here. The frequency (25) in the nonlinear approximation (24) taking into account (23) is in the form

$$\hat{\omega}(k) = \omega(k) - \frac{1}{4}\Big(a_f^2(k) - b_f^2(k) + 2\mathrm{i}a_f(k)b_f(k)\Big)\frac{J(k,\omega(k))}{\omega(k)}. \tag{30}$$

Expression (30) imposes constraints on the envelope amplitude where we observe that an imaginary term appears in the exponent. The presence of this term leads to instability. This can be avoided by setting to zero either the imaginary $b_f(k)$ or real $a_f(k)$ part of a, which can be easily achieved by using an even or odd function $f(x)$, respectively.

Special cases. It is evident that in the case of an even function $f(x)$ (below, the index 'ev' indicates the values corresponding to this case), we can transition to simpler expressions with integrals over $(0,+\infty)$

$$a_f^{ev}(k) = \frac{1}{\pi}\int_0^{+\infty} f(\xi)\cos k\xi d\xi, \quad b_f^{ev}(k) \equiv 0,$$

in the linear approximation

$$a_{lin}^{ev}(k) = \frac{1}{4\pi}\int_0^{+\infty} f(\xi)\cos k\xi d\xi,$$

$$\eta_{lin}^{ev}(x,t) = \int_0^{+\infty} a_f^{ev}(k)\times \times\Big(\cos(kx - \omega(k)t) + \cos(kx + \omega(k)t)\Big)dk,$$

and in the nonlinear approximation

$$\eta^{ev}(x,t) = \eta_1^{ev}(x,t) + \eta_2^{ev}(x,t),$$

where

$$\eta_1^{ev}(x,t) = \int_0^{+\infty} a_f^{ev}(k)\times$$
$$\times\left(\cos(kx-\hat{\omega}^{ev}(k)t)+\cos(kx+\hat{\omega}^{ev}(k)t)\right)dk,$$
$$\eta_2^{ev}(x,t) = \frac{1}{4}\int_0^{+\infty} \Lambda(k,\omega(k))(a_f^{ev})^2(k)\times$$
$$\times\left(\cos 2(kx-\hat{\omega}^{ev}(k)t)+\cos 2(kx+\hat{\omega}^{ev}(k)t)\right)dk,$$
$$\hat{\omega}^{ev}(k)=\omega(k)-(a^{ev})^2\frac{J(k,\omega(k))}{\omega(k)},\ a^{ev} = \frac{a_f^{ev}(k)}{2},$$
$$A_\pm^{ev}(t,k) = \frac{1}{2}a^{ev}\exp\left(\pm \mathrm{i}(a^{ev})^2\frac{J(k,\pm\omega(k))}{\omega(k)}t\right).$$

A similar result can be easily obtained for another specific case when the function $f(x)$ is odd.

Conclusion and further developments

Linear and weakly nonlinear integral expressions for waves propagating between two liquid half-spaces, induced by the initial deviation of the contact surface, have been derived. A limitation on the initial position of the contact surface deviation within the model has been noted, stemming from the characteristics of the solution obtained using the method of multiscale expansions. The prospect of this study lies in the potential to obtain solutions for various geometric and physical properties using symbolic and numerical computation methods. In particular, based on the solutions presented here and the Benjamin-Feir stability condition, it will be possible to derive the conditions for the emergence of rogue waves.

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Авраменко О. В.

ВІДХИЛЕННЯ ПОВЕРХНІ КОНТАКТУ ДВОХ РІДКИХ НАПІВПРОСТОРІВ З ПОВЕРХНЕВИМ НАТЯГОМ: БАГАТОМАСШТАБНИЙ ПІДХІД

Ця стаття присвячена дослідженню відхилення поверхні контакту між двома напівнескінченними рідинами під впливом сил поверхневого натягу та гравітації з використанням багатомасштабного аналізу. Початково-крайова задача базується на ключових безрозмірних параметрах, зокрема на відношенні густин і коефіцієнті поверхневого натягу, для опису генерації та поширення хвильових пакетів уздовж поверхні контакту. За допомогою слабко нелінійної моделі досліджують початкові відхилення поверхні контакту, що дозволяє отримати інтегральні розв'язки для як лінійного, так і нелінійного наближень. Лінійне наближення описує основну структуру прямої та зворотної хвиль, тоді як нелінійні поправки враховують ефекти вищого порядку, які виводяться за допомогою багатомасштабних розкладів. Ці поправки характеризують еволюцію обвідної хвильового пакета, виявляючи взаємодію між дисперсією, нелінійністю та поверхневим натягом. Надаються інтегральні вирази для лінійних і нелінійних розв'язків, зокрема таких, що демонструють роль парних і непарних початкових відхилень поверхні контакту. Порівняння між лінійним і нелінійним наближеннями підкреслюють їх взаємозв'язок. Лінійна модель встановлює основну динаміку хвиль, тоді як нелінійні члени додають гармоніки вищого порядку, уточнюючи розв'язки і дозволяючи проводити аналіз стійкості. Ці результати виявляють суттєві внески від гармонік вищого порядку у визначення динаміки поверхні контакту. Крім того, у дослідженні розглянуто умови, за яких нелінійна обвідна залишається стійкою, зокрема обмеження на початкові амплітуди, щоб запобігти виникненню нестійкості. Дослідження відкриває нові перспективи для подальшого аналізу стійкості та динаміки хвиль на межі поділу рідин за допомогою символьних обчислень. Потенційні застосування передбачають подальше вивчення поведінки хвиль за різних геометричних параметрів системи та властивостей рідин. Отримані результати сприяють розвитку моделювання гідродинамічних хвиль і закладають основу для подальших досліджень у цій галузі.

Ключові слова: внутрішні хвилі, початково-крайова задача, багатомасштабні розвинення, поверхневий натяг.

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